Quantum combinatorial algorithms

Team 14

Numerical Linear Algebra, Skoltech

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Success of Combinatorial model

$$\text{Model}=\begin{pmatrix} \textit{graph} \\ \textit{object}_1 \end{pmatrix}+\begin{pmatrix} \textit{site} \\ \textit{object}_2 \end{pmatrix}$$

- Ising-like models$=\begin{pmatrix} \textit{structure} \\ \textit{bond} \end{pmatrix}+\begin{pmatrix} \textit{charge} \\ \textit{spin} \end{pmatrix}$ describes magnetism.
- Hopfield model$=\begin{pmatrix} \textit{neural} \\ \textit{network} \end{pmatrix}+\begin{pmatrix} \textit{neuron} \\ \textit{firing} \end{pmatrix}$ comp.neuroscience.
- Tensor network describes approximate computational model.

$\rightarrow$Phase transition

Combinatorial problems are NP-hard.

Big question

How far combinatorial(approximation) algorithms could go?

Combinatorial object

- Ising model=Chromatic number[Welsh and Merino, 2000]
  Ising model is NP hard. Graph polynomial is hard.
  Zero limit of Ising model is finding coloring from the polynomial.
- Tensor network
  Most tensor problems are NP-hard[Hillar and Lim, 2013]
  Tensor network algorithms are also complexity hard
  APPROXIMATION

$\rightarrow$ Tensor network Renormalization Group

Quantum chromatic number

Graph coloring game(Classical chromatic number χ)

Alice wins the game if she can *graph-color* without interaction.

Quantum graph coloring game(χ_q)

Graph coloring game+Quantum pseudotelepathy(entanglement)

- $\zeta \leq \chi_q \leq \chi$, $minV(G_{known}) = 1609^{Avis2006}, 18^{Cameron2007}$

Problem statement 1

Is there graph smaller than 18 whose quantum and classical chromatic numbers are different?

Thus spoke Ivan, Graph coloring is

, with some optimization methods,

- Eigenvalues of Laplacian[Wilf-Hoffman's bound]

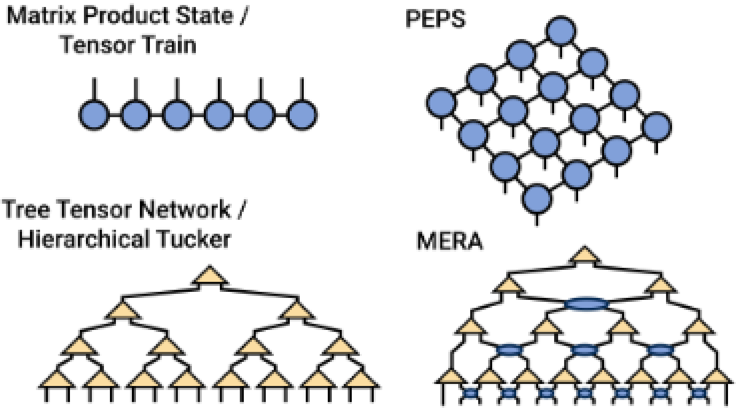
$L = Id - A = (_i = d - \mu_i) \rightarrow d_{ave} \leq \mu_1 = max\frac{x^TAx}{x^Tx} \leq d_{max}$

$_{max}(A) \geq_{max} (A') \geq_{min} (A') \geq_{min} (A)$, $[\mu_1] + 1 \leq \chi(G) \leq \frac{\mu_1 - \mu_n}{-\mu_n}$

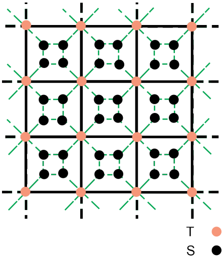
- Defining coloring relation

matrix k coloring, vector k coloring(equivalent)
vector $k + \epsilon$ coloring can be constructed in $O(nlog(\frac{1}{\epsilon})$ with Cholesky factorization
(combinatorial optimization relaxation-maxcut algorithms-goesmanwiliam)

And thus spoke Ivan, Tensor RG is



Square lattice



- SVD decomposition
- Reduce rank
- Merge
- Calculate energy

Square lattice[Cook,MIT project15]

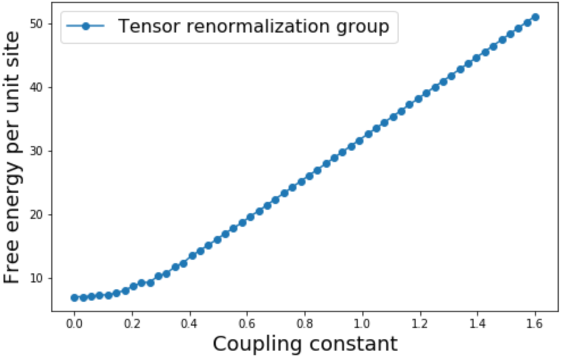
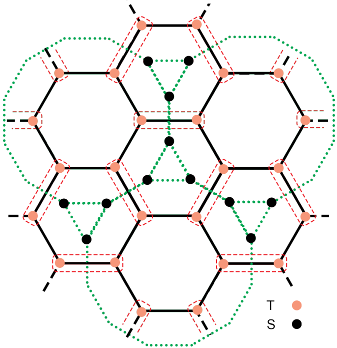


Figure 1: Square lattice tensor network renormalization group

Graph coloring

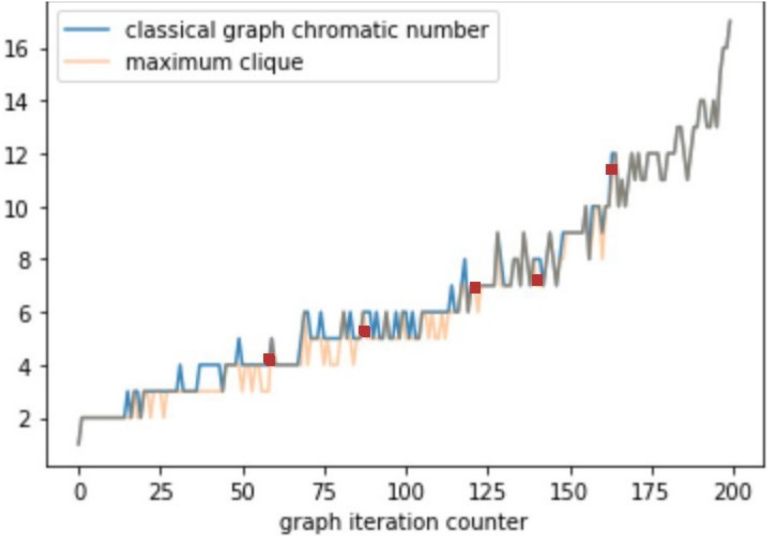
- $O(3^N)$: *coloring*, $O(2^N N^2)$: *MAXclique*
- Greedy heuristic algorithm with logarithmic error bound
  - Set cover based approach
  - Bipartite coloring optimization
  - Local search

TRG on Honeycomb lattice



- Geometrical replacement
- SVD decomposition
- Reduce rank
- Merge
- Calculate energy

Chromatic number results(18)



Tensor network renormalization result (Honeycomb lattice)

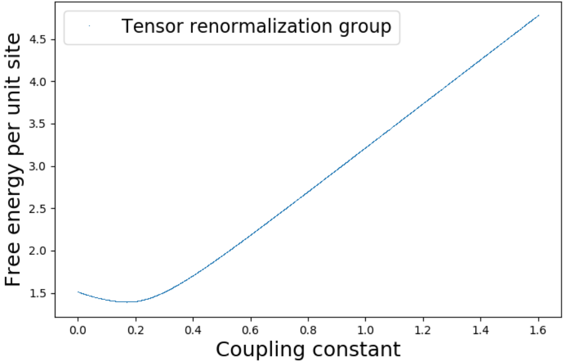
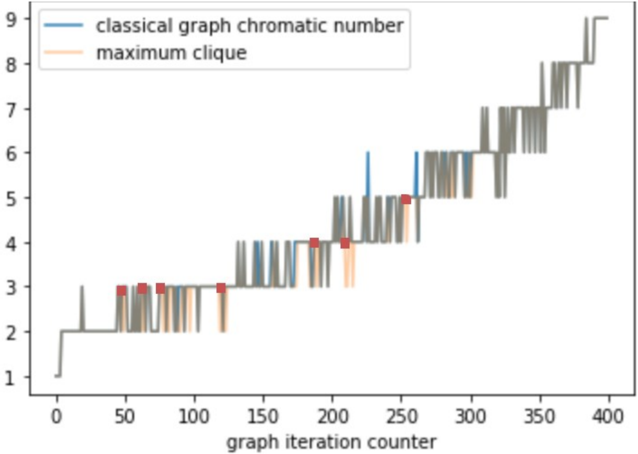


Figure 2: Honeycomb latice tensor network renormalization group

Conjecture

Additional NLA project(Ivan's problem)

There exist $n^{\in \mathbf{N}}$ such that $\forall G, V(G) \leq n \rightarrow |\zeta(G) - \chi(G)| = 1$



Real Discussion

- What is the polynomial extension of quantum chromatic number?
  - What is the complexity class?
  - How does it related to physical phenomenon?
- Can tensor network explain (quantum) communication complexity?
  - Nonlocal game(=graph coloring game)
  - Bell's inequality(Tsirelson's conjecture)

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