# Improved Fixed-Rank Nyström Approximation via QR Decomposition: Practical and Theoretical Aspects

(Farhad Pourkamali-Anaraki and Stephen Becker)

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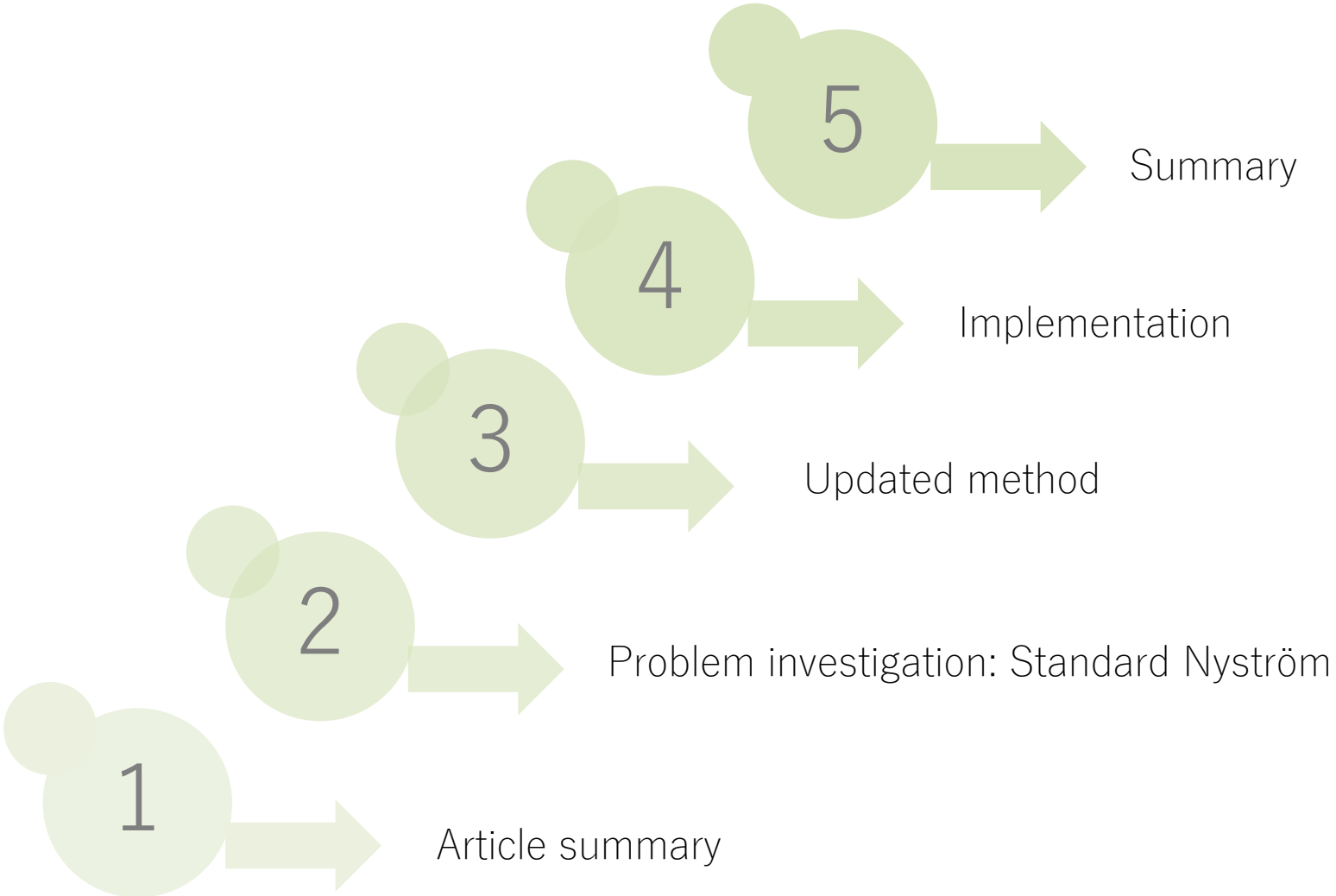
# Presentation plan

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# 1 Article statements

The Nystrom method is a popular technique for computing fixed-rank approximations of large kernel matrices using a small number of landmark points

The Standard Nystrom method possesses poor performance and lack of theoretical guarantees

To improve approximation The Modified Nystrom approximation is proposed

To demonstrate the advantages of the modified method theoretical analysis and numerical experiments are provided

# 2 Standard Nyström Method vs. Modified

**Input:** data set X, m landmark points Z, kernel function k, target rank r (r<m)

$$C_{i,j} = k(x_i, z_j) \qquad W_{i,j} = k(z_i, z_j)$$

$$EVD: W = V \Sigma V^T$$

$$L^{nys} = C V_r (\Sigma_r^{\dagger})^{1/2}$$

$$EVD: (L^{nys})^T L^{nys} = \tilde{V} \tilde{\Sigma} \tilde{V}^T$$

$$\hat{U}_r^{nys} = L^{nys} \tilde{V} (\tilde{\Sigma}^{\dagger})^{1/2} \qquad \tilde{\Lambda}_r^{nys} = \tilde{\Sigma}$$

$$QR: C = QR$$

$$EVD: RW^{\dagger}R^T = \tilde{V}' \tilde{\Sigma}' \tilde{V}'^T$$

$$\hat{U}_r^{opt} = QV'_r \qquad \tilde{\Lambda}_r^{opt} = \Sigma'_r$$

**Output**: estimates of r leading eigenvectors and eigenvalues of kernel matrix

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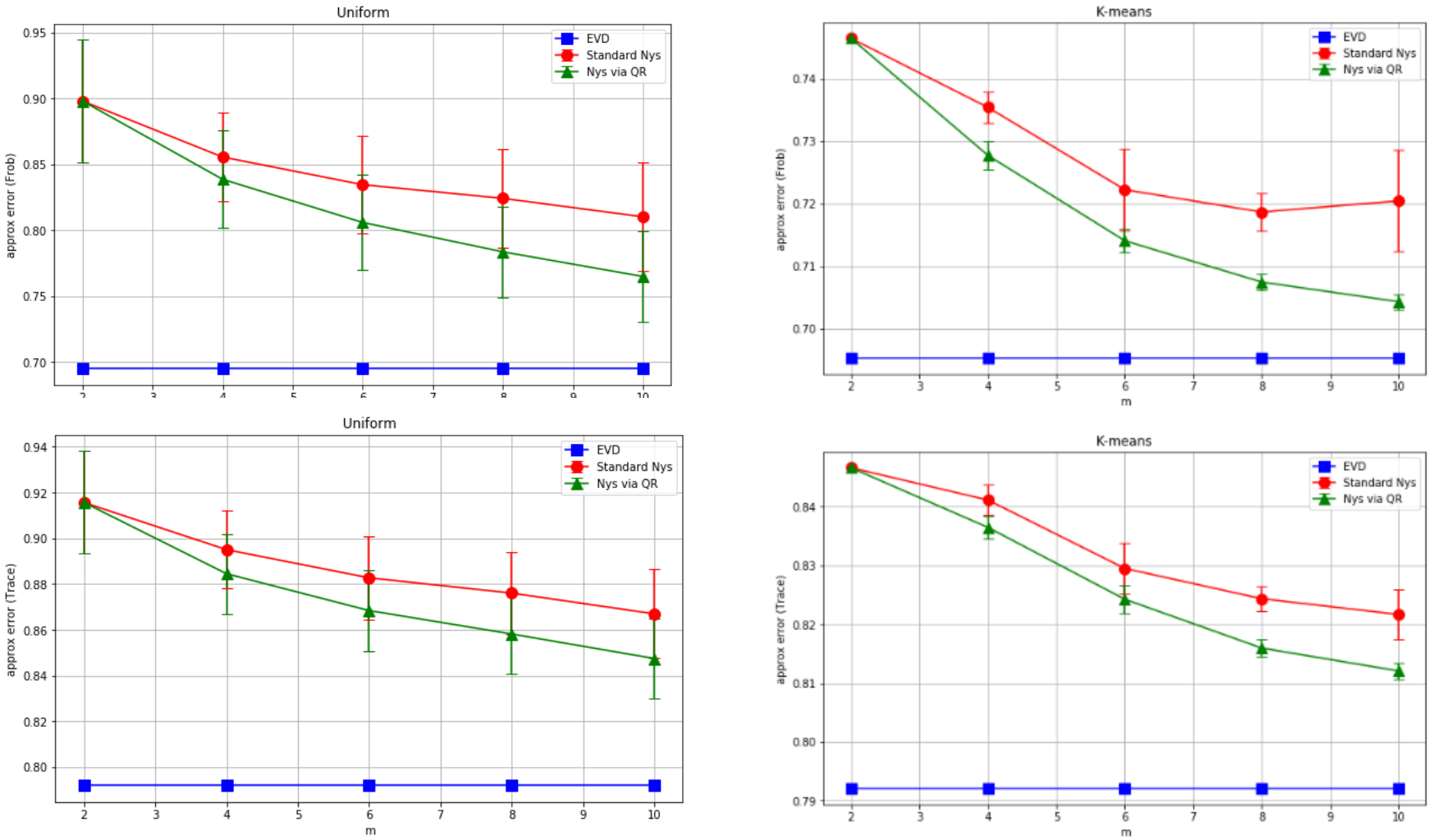
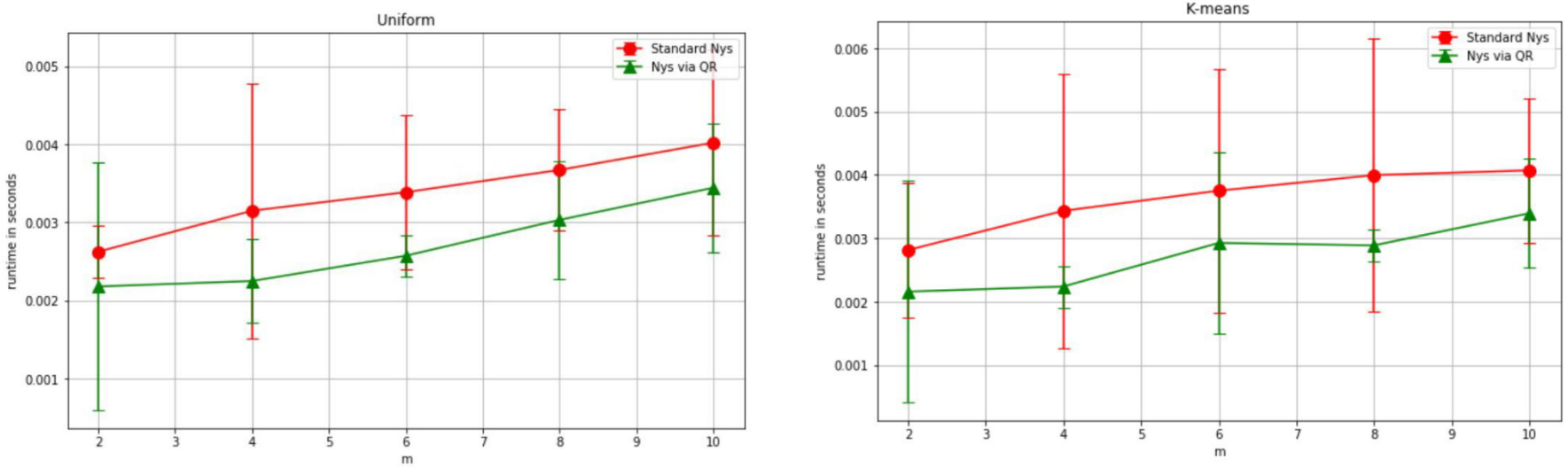
# 3 Implementation




Average errors (Frobenius and Trace norms) for Uniform (left) and K-means (right) algorithms

# 3 Implementation




Runtime for Uniform (left) and K-means (right) landmark points choice algorithms

# 4 Summary

A modified technique for the important process of rank reduction in the Nystrom method has been presented

The modified method provides improved fixed-rank approximations compared to standard Nyström

The quality of fixed-rank approximations generated via the modified method improves as the number of landmark points increases

Illustrative numerical experiments shows benefits of the modified Nystrom method over standard Nystrom method

For future development other decompositions might be performed (Skeleton, SVD etc.)

More data sets can be implemented

# 5 Reference list


1. Pourkamali-Anaraki F., Becker S., 2017. Improved Fixed-Rank Nyström Approximation via QR Decomposition: Practical and Theoretical Aspects. CU Boulder, USA. arXiv reprint: *arXiv:1708.03218v1*.

2. Chang C., Lin C., 2011. LIBSVM: A library for support vector machines. ACM transaction on Intelligent Systems and Technology, vol. 2, no. 3, pp. 27:1-27:27.

# Thank you for your attention!

Questions are welcome