

# Basics of NLA

NLA course

2018

If you do not know answers to the following questions or can not solve the following problems, you will likely go to re-examination.

## 1. Questions

1. Floating vs. fixed point representation of numbers.
2. Definitions of vector and matrix norms. Basic norms: p-norms (vector and matrix), Frobenius norm.
3. Complexity of basic linear algebra operations: e.g. matrix-vector, matrix-matrix products.
4. Definition of Hermitian and unitary (symmetric, orthogonal) matrices. Properties of their eigenvalues.
5. Definition of positive definite matrix.
6. Normal matrices and their properties.
7. Definition, existence and uniqueness of basic matrix decompositions: LU, QR, Cholesky, Schur, SVD, skeleton, eigendecomposition. Computational complexity of these decompositions.
8. Definition of condition number.
9. Definitions of key matrices: Fourier, permutation, Householder, Givens, Hessenberg, triangular, Toeplitz, circulant.
10. Formulation of Eckart-Young theorem.
11. Formulation of the QR algorithm.
12. Least-squares problem. Definition of a pseudoinverse.
13. Power method and how it converges.
14. CSR format.
15. Richardson iteration. Optimal parameter.
16. Krylov subspace. Idea of main Krylov methods: Lanczos, Arnoldi, CG, MINRES, GMRES, bicgstab. Differences between these methods and when to apply them.
17. Idea of ILU preconditioning.
18. Fast Fourier transform and how it helps to multiply vector by Toeplitz matrix fast.

## 2. Problems

1. Find at least one eigenvalue and eigenvector pair of matrix  $A = \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ . How many eigenvalues are there?
2. Find singular values of matrix  $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
3. Write down formula for Householder reflection matrix
4. Write down formula for Givens rotation matrix
5. Show that if  $A$  is skew-Hermitian ( $A^* = -A$ ), then  $I - A$  is nonsingular
6. Prove that  $\|xy^\top\|_F = \|xy^\top\|_2 = \|x\|_2\|y\|_2$  for any  $x, y \in \mathbb{C}^n$
7. Solve  $\frac{\partial}{\partial a} x^\top a$
8. Solve  $\frac{\partial}{\partial A} \text{tr} A^\top B$
9. Show that if matrix is triangular and unitary, then it is diagonal.
10. Eigenvalues of a unitary matrix?